The Sixth International Symposium on Fuzzy Sets

ISFS 2025

Katowice, Poland
May 23 – 25, 2025

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Abstracts of the 6th International Symposium on Fuzzy Sets (ISFS 2025)

Editors: Michał Baczyński, Przemysław Grzegorzewski, Katarzyna Miś

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Michał Baczyński, Przemysław Grzegorzewski, Katarzyna Miś
Abstracts of the 6th International Symposium on Fuzzy Sets (ISFS 2025)

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Introduction

Sixty years ago, Lotfi A. Zadeh published a concise paper entitled "Fuzzy Sets" (Information and Control, 8 (1965), 338–353), in which he introduced a new perspective on perceiving and modeling uncertainty. This work, which laid the foundations for a revolution in logic and set theory, turned out to be groundbreaking. It sparked a scientific and technological revolution whose impact is hard to overestimate — resulting in hundreds of thousands of academic papers, patents, technical applications, and more (as of now, the paper has been cited over 160,000 times).

Since the inception of fuzzy set research, Polish scholars have played a significant role in developing the theory and its applications, actively collaborating with researchers from numerous European countries and beyond. This cooperation has led not only to joint publications and grants but also to academic events — seminars, conferences, workshops, and so on.

One initiative that emerged during this period is the International Symposium on Fuzzy Sets (ISFS). In its early stages, the ISFS was a local event, bringing together researchers mainly from Poland and Slovakia, and organized alternately in both countries. Over the years, the symposium began attracting growing numbers of scientists from the Czech Republic, Spain, Italy, and other countries. After the establishment of the Polish Society for Fuzzy Sets (POLFUZZ), it was decided that the former seminar would be transformed into an international conference, held biennially under the patronage of POLFUZZ, while maintaining continuity in the numbering of editions. Thus, in 2023, the 5th ISFS took place in Rzeszów, and the 6th edition of the conference — celebrating the 60th anniversary of fuzzy sets — will be held in 2025 in Katowice.

The organizers of the ISFS aim not only to create a platform for scientific discussion and exchange of experiences but also to promote the legacy initiated by Lotfi Zadeh and his successors. Therefore, each edition of the ISFS has featured distinguished scholars delivering plenary talks, and special attention has been given to encouraging the participation of students and PhD candidates. A new initiative accompanying this year's ISFS is the awards gala of the "National Competition of the Polish Society for Fuzzy Sets for the Best Thesis on Fuzzy Set Theory and Applications".

This volume contains a collection of abstracts of the presentations delivered during this year's conference, held at the University of Silesia in Katowice from May 23–25, 2025. We hope that — just like in previous years — it will bring numerous scientific outcomes, foster the renewal and establishment of contacts among researchers from various institutions around the world, and remain a positive memory for all participants.

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Plenary Lectures

Gabriella Casalino (University of Bari Aldo Moro, Italy)

Lucian Coroianu (University of Oradea, Romania)

József Dombi (University of Szeged, Hungary)

Marek Reformat (University of Alberta, Canada)

Explaining Evolving Data

Gabriella Casalino

University of Bari Aldo Moro, Italy

Digitalization has transformed our daily lives, leading to the continuous collection of vast amounts of data from various interconnected devices. This evolving data serves as a crucial source of information that must be processed efficiently and in almost real-time to extract meaningful insights. However, the dynamic nature of data streams poses significant challenges, requiring specialized algorithms that can adapt to changes, process information dynamically, and update models incrementally without retraining from scratch.

One major obstacle in handling evolving data is the availability of labeled examples. Obtaining labeled data is costly, time-consuming, or even infeasible in many real-world scenarios. Semi-supervised learning approaches mitigate this issue by enhancing learning performance by leveraging labeled data, when available, and the underlying geometric structure of the unlabeled data.

Beyond efficient data processing, explainability is fundamental in critical domains such as healthcare and education, where decisions must be transparent, interpretable, and justifiable. Fuzzy logic provides a compelling framework for addressing both the learning and explainability aspects of evolving data. It enables handling uncertainty inherent in data while offering human-interpretable explanations in natural language, bridging the gap between complex computational models and user understanding.

This talk explores how a prototype-based evolving fuzzy algorithm can be used to derive meaningful explanations in dynamic environments. The algorithm continuously adapts to new data while maintaining interpretability and fostering trust in AI-driven decision-making processes.

Approximation methods in fuzzy analysis

Lucian Coroianu

University of Oradea, Romania

In this talk I will present some of the main results concerning various approximation problems in the theory of fuzzy numbers and of OWA operators. There are two principal ways to approximate fuzzy numbers. The first method is the one in which one approximates the membership function. The suitable metric here is the Chebyshev metric. We will present here approximations based on the Bernstein max-product operator and then using the extended inverse F-transform including an approach where the components of the direct F-transform are obtained by minimizations in the L_p - norm, $1 \leq p < \infty$. These non-linear approaches have the advantage that they preserve better the shape of a fuzzy number than the ones based on linear approximation operators. The second method to approximate fuzzy numbers is when we work with the level sets representations. We will recall some results concerning the approximations by trapezoidal and more generally, piecewise linear fuzzy numbers by L_2-type metrics including also results on Lipschitz continuity. This second problem can be reduced to the study of quadratic programs and the techniques used to solve such problems are inspirational for problems that apparently belong to other areas. However, the goal is the same, that is, the finding of algorithms that can be implemented easily on computers. Such problems will be discussed in this presentation insisting on the constrained OWA aggregation problem but also on some methods to find optimal OWA weights under some constraints. A common feature in all these studies is that some of the results obtained in problems that belong purely to the domain of optimization (or more generally convex analysis) or approximation theory can be used to solve (or improve, as for example the obtaining of sharper Lipschitz constants) concrete problems in fuzzy sets theory. But also the reverse is true, problems that are relevant in fuzzy sets theory can lead to interesting problems in optimization or approximation theory. This talk will emphasize some of these aspects too.

On the fuzzy concept

József Dombi

University of Szeged, Hungary

We can define a variety of different approaches, which we call fuzzy. There are many different operator classes, in the literature (T-norms, T-conorms, Ling-theorem). These operators are generalizations of conjunctive and disjunctive operators. We also have many types of negation operators (Sugeno, Hamacher, Yager). The implication operators also form a new class of operators. Many books have been published on this topic. But it is still not enough as other problems arise related to the definition of operators, such as equivalence and symmetric difference operators. The unary operators also form a large class. All these above-mentioned problems are related to the concept of continuous-valued logic.

We can speak of a fuzzy concept if we have some knowledge about the arguments of the operators. Another key aspect in fuzzy theory is how we define the membership functions. Nowadays, more than twenty different types exist.

We can choose operators with different membership functions. The suitability of our particular choice depends on the use case.

In this lecture, the models depend on a single generator function of a single operator system and each operator has only one generator function. The membership function represents either a soft inequality or equality. This membership system is consistent with the logic of ours. We will illustrate what we mean with concrete examples and theorems.

State Assessment in Complex Systems: An Evidence Theory Approach to Multi-granular Data Integration

Marek Reformat

University of Alberta, Canada

Complex real-world systems present significant analytical challenges due to their multi-level hierarchical structures and interdependent subsystems. These systems are characterized by intricate relationships where each subsystem's state depends on numerous inputs and the conditions of interconnected components. A fundamental challenge lies in precisely defining these subsystem states - definitions that inherently contain uncertainty and are often articulated by domain experts using information granules (such as linguistic descriptors or interval representations) to manage areas of inexactness.

Current methodologies fail to comprehensively address the multiple uncertainty sources while providing reliable state assessments across system levels. This paper presents an innovative approach for determining local and global states in hierarchical multi-component systems under uncertain conditions. Our methodology leverages Evidence Theory principles and introduces a new technique for evaluating uncertain target satisfaction, where the targets themselves are granular state definitions.

The proposed framework systematically manages three critical sources of uncertainty: measurement imprecision in subsystem inputs, propagated uncertainty in state determinations based on interconnected subsystems, and the inherent imprecision within expert-defined state descriptions. By calculating satisfaction degrees between inputs and imprecise state definitions, we can identify the most probable state of each subsystem and, consequently, of the entire system.

We provide a detailed exposition of our methodology alongside a comprehensive case study demonstrating the practical application of our approach in determining system states despite imprecise subsystem state definitions. This work addresses a significant gap in complex systems analysis under uncertainty conditions.

Abstracts

Conditional and permutation-based OWA operator

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Data aggregation using nonadditive operators is crucial in various applications as it considers interactions between data. Already in 1925, Giuseppe Vitali introduced the idea of aggregation (specifically integration) with respect to a not necessarily additive measure [5]. This approach was later developed by Gustave Choquet in his pioneering work [3] on the theory of capacities in 1954. This laid the foundation for the Choquet integral, which falls into the class of nonadditive (or fuzzy) integrals and finds applications in various fields. In 1988, Ronald Yager in [6] introduced the OWA operator for data aggregation. It is defined for $\mathbf{x} \in [0, \infty)^n$ as

$$\mathrm{OWA}_{\mathbf{w}}(\mathbf{x}) = \sum_{i=1}^{n} w_i \cdot x_{\phi^{\uparrow}(i)},$$

where $\mathbf{w}$ is a weight vector with $w_1 + \cdots + w_n = 1$, and $\phi^{\uparrow}$ is a permutation of basic set that reorders the components of $\mathbf{x}$ in nondecreasing order. This construction allows for the consideration of not only the input values but also their order and weights, which reflect priorities. Thanks to these, the OWA operator has become a valuable tool for decision-making processes. As shown by Murofushi in 1993 [4], the OWA operator is a special case of the Choquet integral, if a monotone measure is symmetric. This finding introduced a new approach to deriving operators from nonadditive integrals by simply replacing the monotone measure with weights.

The literature features many generalizations, whether of the Choquet integral or the OWA operator. We can observe the constructions where the permutation of the basic set is derived from the input vector, or it can be chosen arbitrarily in advance. Furthermore, the permutation of the basic set may not be related to the input vector but can pertain to the aggregation of its components (e.g., the TOWA operator, in which the aggregation is performed by triangular norms.).

In [1], we introduced a novel operator that covers several Choquet-like operators. Our approach is based on generalizing the permutation through the concept of a class of permutation pairs, where each vector is associated with an arbitrary preselected permutation of the basic set. The second significant element of the construction is the concept of a conditional aggregation operator introduced in [2], which generalizes aggregation functions. Formally, a map $\mathsf{A}(\cdot|B)\colon [0,\infty)^n \to [0,\infty)$ is called a conditional aggregation operator w.r.t. a set $B \in 2^{[n]} \setminus \{\emptyset\}$, if

(i) $\mathsf{A}(\mathbf{x}|B) \leq \mathsf{A}(\mathbf{y}|B)$ for any $\mathbf{x}, \mathbf{y} \in [0,\infty)^n$ such that $x_i \leq y_i$ for any $i \in B$,
(ii) $\mathsf{A}(\mathbf{1}_{B^c}|B) = 0$.

In the contribution, we define a novel OWA operator. Following the construction in [4] and Murofushi's aforementioned observation, substituting the monotone measure with weights, we define the conditional and permutation-based OWA operator. We highlight its properties and potential applications in noise reduction as part of image processing.

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Applications of the Choquet-like integrals based on multidimensional input

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We show that any method of decision making under probabilistic uncertainty based on expected value, variance or covariance can be adapted to the case of uncertainty described by a monotone measure using the new extension of the Choquet integral to the multi-valued framework [3], [4], [5]. We also introduce a method of defining the average value of a finite sequence of vectors, which occurs, for example, in the valuation of swap contracts. We demonstrate the procedure of aggregating a vector of any type of input (such as intervals, vectors, etc.), using the generalized Choquet integral with respect to an admissible order, getting the desired type of output [1]. For the introduced operators, further potential fields of applications will be proposed and their connections to the existing operators will be examined [2]. We also present a collaboration opportunities.

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Scientometric indices: the compensation of different research careers

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Hirsch's $\mathfrak{h}$-index [6], as well as Egghe's $\mathfrak{g}$-index [5],

$$\mathfrak{h}(\mathbf{x}) = \max_k\{k \wedge x_k\}, \quad \mathfrak{g}(\mathbf{x}) = \max\left\{k \in [n] : \sum_{i=1}^{k} x_i \geq k^2\right\},$$

are widely recognized tools for assessing scientists based on their publication records. Here, the vector $\mathbf{x} = \{x_1, \ldots, x_n\}$ represents a *scientific record*, where $x_i \in \mathbb{N} \cup \{0\}$ for any $i \in [n]$ and $x_1 \geq \cdots \geq x_n$. Both indices offer advantages, primarily their computational simplicity, but also have well-documented limitations, as discussed in [1]. While an integral representation of the $\mathfrak{h}$-index was introduced in [10], a similar formulation for the $\mathfrak{g}$-index has not yet been established. In this work, we present an operator-based expression for the $\mathfrak{g}$-index and propose a unified operator framework for both indices. This formulation leverages fuzzy measures along with (conditional) aggregation operators—specifically, the minimum and summation—providing a more general perspective on these bibliometric measures [2].

This theoretical foundation enables a more meaningful comparison of scientists with different research trajectories. One of the key limitations of the $\mathfrak{h}$-index and $\mathfrak{g}$-index—their inability to account for career length and research dynamics—has been addressed in previous works such as [7], [8] and [9]. Our contribution builds upon Stupňanová's approach introduced in [9]. On one hand, we analyze the mathematical foundations of the established time-dependent indices $\mathfrak{h}_T$, $\mathfrak{h}^T$, $\mathfrak{h}_T^T$, while also exploring their practical interpretations. On the other hand, we focus on potential temporal modifications of the $\mathfrak{g}$-index, as discussed in the recent paper [3].

We propose two distinct approaches:

(i) a straightforward adaptation of Stupňanová's method, originally designed for the $\mathfrak{h}$-index. While intuitive, this approach does not yield optimal results for the $\mathfrak{g}$-index;

(ii) a refined approach leveraging the already generalized $\mathfrak{g}$-index in the form presented in [4], which intriguingly allows for a decrease in the index value over time. This possibility of decline—typically absent in conventional bibliometric indices—addresses a common critique that an index should not be strictly non-decreasing [1].

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Is there a link between Reichenbach implication and Reichenbach's common cause principle?

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Reichenbach's common cause principle [6] says that there is no correlation without causation. In mathematical formalism, it is formulated as follows: If two events a, b at a state (=probability measure) s satisfy

$$s(a \wedge b) > s(a) \cdot s(b)\,, \tag{1}$$

there should be an event c (*common cause*) satisfying

$$\begin{aligned} s(a \wedge b|c) &= s(a|c)\, s(b|c)\,, \\ s(a \wedge b|c') &= s(a|c')\, s(b|c')\,, \\ s(a|c) &> s(a|c')\,, \\ s(b|c) &> s(b|c')\,. \end{aligned}$$

In [5], Hans Reichenbach studied a many-valued logic with the interpretation of implication given by

$$\alpha \rightarrow \beta = 1 - \alpha + \alpha\,\beta\,,$$

so-called *Reichenbach implication* [1].

A natural question arises whether there is any link between these two Reichenbach's contributions. An AI answer is negative [3]:

> *...aside from the historical link to Hans Reichenbach's work, the Reichenbach S-implication and Reichenbach's Common Cause Principle remain separate and unrelated in both theory and application.*

Nevertheless, we reformulated the Reichenbach's common cause principle using the Reichenbach implication and it seems rather natural in this interpretation. Two events a, b are independent in the state s iff the probability of its implication, $a \Rightarrow b$ (where $\Rightarrow$ denotes the implication between *events*, not truth degrees) satisfies $s(a \Rightarrow b) = s(a) \rightarrow s(b)$, and similarly for (1). Then the Reichenbach's common cause principle can be equivalently reformulated as follows: If two events a, b at a state s satisfy

$$s(a \Rightarrow b) > s(a) \rightarrow s(b) \,,$$

there should be an event c (*common cause*) satisfying

$$\begin{aligned} s(a \Rightarrow b|c) &= s(a|c) \rightarrow s(b|c) \,, \\ s(a \Rightarrow b|c') &= s(a|c') \rightarrow s(b|c') \,, \\ s(a \Rightarrow c) &> s(a) \rightarrow s(c) \,, \\ s(b \Rightarrow c) &> s(b) \rightarrow s(c) \,. \end{aligned}$$

The existence of a common cause is the main topic of the monograph [4]. In our previous work [2,7], we studied the existence of common cause (which appears “rare”) and the possibility of embedding in a larger logic where all common causes exist (which appears “common”). We hope that the new formulation of the Reichenbach's common cause principle could contribute to its better understanding and further simplification and extension of the related results.

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Comparative study of fuzzy clustering techniques

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Fuzzy clustering methods form an important family of clustering algorithms, with their origins tracing back to the 1970s, as demonstrated in the work of Dunn [4]. Among the most notable techniques is the fuzzy c-means algorithm, introduced by Bezdek in the 1980s [1, 2]. Since then, many other compelling methods have emerged, such as the more generalized possibilistic clustering approach [5] along with various modifications. Additionally, fuzzy theory has been applied to hierarchical clustering methods, allowing for more flexible partitioning, as seen in studies like [3, 7]. More recently, fuzzy clustering ensemble methods have attracted significant research interest, as they combine multiple fuzzy partitions to produce more robust results, see e.g., [6, 8]. This paper presents a comparative study that evaluates a representative set of fuzzy clustering methods from a practical perspective. We will use a diverse range of benchmark datasets and conduct numerical experiments to empirically assess the strengths and weaknesses of these methods, offering valuable insights into their applicability and, hopefully, providing practical guidance on how to choose the most appropriate method for specific applications.

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On the convergence of interval valued observables

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The aim of this contribution is to formulate various types of convergence for a sequence of interval valued observables. We will work with the family of interval valued events

$$\begin{aligned}\mathcal{K} = \{(\pi_C, \rho_C)\,;\, \pi_C \le \rho_C,\ \pi_C, \rho_C : \Omega \to [0,1] \\ \text{are } \mathcal{S} - \text{measurable functions}\}\end{aligned}$$

with the operations and relation

$$\begin{aligned}\mathbf{C} \preceq \mathbf{D} &\Leftrightarrow \pi_C \le \pi_D, \rho_C \le \rho_D \\ \mathbf{C}\widehat{\oplus}\mathbf{D} &= \big((\pi_C + \pi_D) \wedge 1_\Omega, (\rho_C + \rho_D) \wedge 1_\Omega\big) \\ \mathbf{C}\widehat{\odot}\mathbf{D} &= \big((\pi_C + \pi_D - 1_\Omega) \vee 0_\Omega, (\rho_C + \rho_D - 1_\Omega) \vee 0_\Omega\big),\end{aligned}$$

which was introduced by L.A. Zadeh in [9].

In [1] we studied convergence in distribution of interval valued observables and we proved a variation of Fisher-Tippett-Gnedenko theorem and the variation of Pickands-Balkema-de Haan theorem for interval valued observables. By an interval valued observable z on $\mathcal{K}$ we understand each mapping $z : \mathcal{B}(R) \to \mathcal{K}$ satisfying the following conditions:

(i) $z(R) = (1_\Omega, 1_\Omega)$, $z(\emptyset) = (0_\Omega, 0_\Omega)$;
(ii) if $A \cap B = \emptyset$, then $z(A)\widehat{\odot}z(B) = (0_\Omega, 0_\Omega)$ and $z(A \cup B) = z(A)\widehat{\oplus}z(B)$;
(iii) if $A_n \nearrow A$, then $z(A_n) \nearrow z(A)$,

see [2].

In this contribution we will study the convergence in measure, the almost everywhere convergence and the almost uniform convergence of interval valued observables. We will prove a variation of Central

Limit Theorem, a variation of Weak Law of Large Numbers, a variation of Strong Law of Large Numbers and a variation of Egorov's theorem for interval valued observables. We were inspired by the various types of convergence in the space of intuitionistic fuzzy events and by the fact that there exists a correspondence between interval valued events and intuitionistic fuzzy events, see [3–8].

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The complement operation of φ-intuitionistic fuzzy sets

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In this contribution, we characterize the negations of φ-intuitionistic fuzzy sets. A strictly increasing bijection φ on the unit interval $[0,1]$ is an automorphism. Each automorphism induces a strong negation [2], i.e., a function $n : [0,1] \to [0,1]$ that is strictly decreasing, continuous, and involutive, given by $n_\varphi(x) = \varphi^{-1}(1 - \varphi(x))$.

A φ-intuitionistic fuzzy set [1] A over a non-empty universe $\mathbf{X}$ is defined as

$$A = \{\langle x, \mu(x), \nu(x)\rangle : (\mu(x), \nu(x)) \in \mathbf{IV}_\varphi, \forall x \in \mathbf{X}\},$$

where $\mathbf{IV}_\varphi$ denotes the set of φ-intuitionistic values associated with the automorphism φ:

$$\mathbf{IV}_\varphi = \{(u, v) : u, v \in [0,1], \varphi(u) + \varphi(v) \leq 1\}.$$

Here, u and v represent the degrees of membership and non-membership of an element $x \in \mathbf{X}$ in the φ-intuitionistic fuzzy set A.

This framework generalizes Atanassov's intuitionistic fuzzy sets [3] and related approaches such as Pythagorean fuzzy sets [4], as they are isomorphic.

Let $\mathbf{iv}_\varphi \subseteq \mathbf{IV}_\varphi$ be a subset of the φ-intuitionistic values. The complement of a φ-intuitionistic fuzzy set A is another φ-intuitionistic fuzzy set A^c given by

$$A^c = \{\langle x, \Phi(\mu(x), \nu(x))\rangle : (\mu(x), \nu(x)) \in \mathbf{IV}_\varphi, \forall x \in \mathbf{X}\},$$

where Φ is the φ-intuitionistic negation, a function $\Phi : \mathbf{IV}_\varphi \to \mathbf{iv}_\varphi$ satisfying:

- $\Phi(1,0) = (0,1)$ and $\Phi(0,1) = (1,0)$,
- $\Phi(u_1, v_1) \geq \Phi(u_2, v_2)$ whenever $(u_1, v_1), (u_2, v_2) \in \mathbf{IV}_\varphi$ satisfy $u_1 \leq u_2$ and $v_1 \geq v_2$.

This negation is representable, meaning that the membership degree of the complement A^c depends only on the non-membership degree of A, and vice versa.

Every φ-intuitionistic negation can be generated by two automorphisms ψ_1, ψ_2 satisfying the inequality

$$\psi_1(x) \geq n_\varphi(\psi_2(n_\varphi(x)))$$

via the formula $\Phi(u,v) = (\psi_1(v), \psi_2(u))$. Moreover, if this inequality holds with equality, i.e.,

$$\psi_1(x) = n_\varphi(\psi_2(n_\varphi(x))),$$

then the codomain of Φ extends to the entire set $\mathbf{IV}_\varphi$. Finally, if the automorphisms satisfy the equalities

$$\psi_1(x) = \varphi^{-1}(n(1-\varphi(x))) \quad \text{and} \quad \psi_2(x) = \varphi^{-1}(1-n(\varphi(x))),$$

where n is an arbitrary strong negation and φ generates $\mathbf{IV}_\varphi$, then Φ becomes involutive, meaning it satisfies

$$\Phi(\Phi(u,v)) = (u,v), \quad \forall (u,v) \in \mathbf{IV}_\varphi.$$

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Combining time series forecasters using operators induced by non-symmetric aggregation functions

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Time series forecasting encompasses a wide variety of methods, each with particular characteristics that have both benefits and limitations. Since choosing the optimal forecaster is usually not a simple task, it is common to apply ensemble approaches, which combine several predictions to improve the performance of the final result [1].

In the context of aggregation functions, one of the most widely used techniques in the last decades is the one based on IOWA operators [6], called Best Yesterday Model. IOWA operators order the data to be aggregated (in this case, the model predictions) by means of an inducing vector, subsequently calculating a weighted mean. A widely used criterion for the choice of the inducing vector is usually an accuracy metric for each of the models in the last timestep.

Classically, the Best Yesterday Model ranks the models according to their accuracy in the last timestep. However, using only the last timestep may be insufficient. In this work, we consider different timesteps to obtain an ordering of the models, as presented in [3].

Considering that the accuracy values are time-dependent, it is convenient to use an aggregation function that is non-symmetric, in order not to break this sequential or temporal relationship. In our case, we have considered pseudo-grouping functions [7], not necessarily symmetric aggregation functions, which have given good results in the fusion of sequential information as for example in text-based Convolutional Neural Networks [2].

There are several construction methods for pseudo-grouping functions, such as from convex sums or from Riemann integration [4] as well as from automorphisms and pseudo-t-conorms [7]. A very important way of constructing them is by means of additive generators, since they construct binary aggregation functions by means of unary functions, thus reducing the computational complexity.

The aggregation of these accuracy values is used to calculate the inducing vector of the IOWA operator, in order to fuse the time series forecasters [5]. The evaluation metrics in time series forecasting has been improved by using pseudo-grouping functions to calculate the inducing vector of the IOWA operator compared to other symmetric aggregation functions.

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Fuzzy Association Rules with Evolving Contexts

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Within the field of Explainable Artificial Intelligence, there is a growing need for approaches that facilitate the generation of explanations in natural language. Such human-centric information granules can support a wide range of applications. This work presents a framework that concentrates on describing time series datasets using natural language. This framework is particularly useful when the interpetationof the considered quantities change over time. Particularly, in the case of economic concepts like *large or small inflation*, the interpretation of the linguistic expressions depends on the current economic situation.

The proposed approach is based on the evaluative linguistic expressions, fuzzy association rules and theory of intermediate quantifiers. At first, we transform time series data using the theory of evaluative linguistic expression [1] into fuzzy sets. The linguistic context w of evaluative linguistic expression is crucial for processing data. For a variable, we usually take a context as triplet $w = \langle v_L, v_S, v_R \rangle$, where v_L is the smallest thinkable value, v_R is the largest thinkable value, and v_S is the common middle value of the variable. Using the context, we are able to create fuzzy sets such as *small*, *big*, etc. The context w w can be determined either by an expert or constructed in a data-driven way. We also distinguish between a static and a dynamic scenarios. For the proposed dynamic data-driven approach, we set the middle point of context v_S for variable in time t, we use some of the previous values to create an autoregressive and moving average model [2], and using this model we predict the next value and this predicted value serves as the middle point of the context v_S for variable in time t. To obtain the left v_L and the right bound v_R, we subtract and add some real numbers. Then, we apply Fuzzy GUHA method [3] for the creation of fuzzy association IF-THEN rules. Let us recall that this method comes out of the GUHA method by Hájek [4].

We demonstrate our framework on an economic case, where the dataset consists of two variables - the sentiment (tone) of the Federal Reserve System and inflation expectation. We transform data using the theory of evaluative linguistic expressions into fuzzy sets. For tone, we use an expert context and for inflation expectations, we use an evolving context. Then, we apply the Fuzzy GUHA method on these fuzzy sets to obtain IF-THEN rules. The resulting explanations can be illustrated through the following example:

*"IF tone is **negative** THEN inflation expectations are **medium**".*

Experiments for real-life data confirmed the validity of the proposed framework.

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Yet another graphical aid to the interpretation and validation of cluster analysis

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Clustering aims at discovering "natural" subsets of observations, e.g., finding out which data points come from a given component of a multidimensional probability distribution mixture, identifying high-density regions of the data domain, or learning about disjoint manifolds from which the inputs are sampled.

Almost 40 years ago, P.J. Rousseeuw in [2] introduced Silhouettes: a tool based on aggregated inter- and intra-cluster distances that facilitates the assessment of the quality of cluster analysis, especially in the case of regularly-shaped clusters. Here we describe a new method in a similar spirit, but based on certain spanning trees [1], which allows us to identify subgroups of different spatial forms.

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F-Integral Driven Aggregation Framework for Message Passing Neural Networks

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Message Passing Neural Networks (MPNNs) have emerged as a powerful framework for processing graph-structured data, where the aggregation of node features plays a critical role in model performance [1]. Traditional aggregation functions in MPNNs, such as sum, mean, or max pooling, often fail to capture complex interactions between messages. While recent works [2] have introduced more generalized fusion operators, they remain limited in scope and do not comprehensively encompass the principal families of aggregation functions used in information fusion [3].

In this work, we propose an alternative approach by incorporating F-integrals as aggregation mechanisms within MPNNs. F-integrals, grounded in capacity theory, provide a more flexible and expressive way to aggregate node features, allowing for the modeling of interactions beyond simple linear combinations [4]. However, their application in large-scale learning problems presents key challenges, particularly in defining, optimizing and computing the fuzzy measures required for the aggregation.

To address these challenges, we introduce a novel fuzzy measure generation and optimization method based on gradient-based learning. Our proposed framework extends the range of possible aggregation operators in MPNNs and opens new directions for information fusion in graph-based learning.

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Time Series Classification with Fuzzy Equivalences and Aggregation Techniques

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Time series classification remains a crucial and challenging task in data analysis. While numerous algorithms have been developed to address this problem, the 1NN classifier stands out as both simple and effective. A comprehensive comparison of distance measures used in time series classification with the 1NN method can be found in [1].

We would like to present an approach to time series classification that involves fuzzy equivalences and aggregation techniques in the 1NN method. In this work, we focus on fuzzy equivalences as defined in [2]. Specifically, we explore various compositions of fuzzy equivalences and aggregation functions as an alternative closeness measure to standard distance metrics. This approach, which has already shown some promise in certain data mining applications (see, e.g., [3–5]), appears to be effective in time series analysis as well.

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Statistical inference about the treatment effect based on imprecise data

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The concept of the treatment effect refers to the difference that can be attributed to a specific intervention, relative to established control conditions. Estimating the treatment effect is often the goal of clinical trials and other experimental studies in social sciences, economics, manufactoring, engineering, and related disciplines.

Recently, both researchers and practitioners have shown increasing interest in inference based on imprecise data (or, more broadly speaking, uncertain data), which is encountered in numerous applications, especially those in which the so-called human factor plays a significant role. To model uncertainty arising from a lack of precision, insufficient information, fluctuations in measured quantities, and similar factors, intervals or fuzzy sets have been successfully employed. Unfortunately, both the sample space consisting of families of closed intervals and families of fuzzy numbers, when equipped with Minkowski operations, do not form linear spaces. This not only creates computational difficulties but also complicates the construction of statistical tools capable of handling the specificity of interval/fuzzy data. Additional challenges stem from the nature of random intervals/fuzzy numbers, particularly from an ontic perspective, which often prevents the application of parametric methods. This highlights the necessity of developing new nonparametric methods and techniques to support statistical inference based on such data.

In our presentation, we will discuss the issue of determining the treatment effect based on imprecise observations, addressing both the challenges related to properly defining this concept and the methods of statistical inference. In our analysis, we will refer to non-standard operations defined in the space of closed intervals and the space of fuzzy numbers (see [4]), as well as nonparametric statistical techniques known from the theory of permutation and bootstrap tests as well as permutation confidence intervals (see [1–3]).

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Triangular norms in some Grothendieck topoi

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Triangular norms (in short t-norms) are special semi-group operations defined on the classical unit interval. Topoi (plural of topos) are categories that can be treated as models of intuitionistic type theory. In this presentation, we will show some results regarding triangular norms in some models of this kind. In particular, the main aim of our presentation is to generalize the theorem that characterizes Archimedean continuous t-norms using multiplicative generators in some Grothendieck topoi.

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Monitoring inhomogeneous complex segmented processes by fuzzy control charts based on Liu's credibility index

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In recent years, we have been faced with streams of data automatically collected by various types of devices. The results of these measurements, represented as series of numerical values, must often be preprocessed for further analysis for various reasons. One such preprocessing method is the aggregation of process data into consecutive segments, which can be seen as a form of granulation. This approach is particularly useful in process description and monitoring. In this paper, we consider data streams that are inhomogeneous and non-stationary. This means that for different periods of observation (segments), the collected data may follow different probability distributions, and the parameters of these distributions may change over time. This type of process, which describes the results of monitoring voice characteristics recorded for psychiatric patients suffering from Bipolar Disorder, was first examined by [3]. In their paper, they considered a possibilistic method of data aggregation. Another aggregation method, based on the concept of p-boxes, was proposed by [5]. In both approaches, the aggregated processes are represented by a series of fuzzy numbers (fuzzy granules) that can be used for identifying the state of the monitored processes. The state of the process can be monitored using various methods, such as, e.g., control charts. Hryniewicz and Kaczmarek-Majer [3, 4] proposed monitoring the aggregated process using simple Shewhart-type control charts designed to monitor certain characteristics of fuzzy numbers. Another approach, used for data aggregated through the concept of p-boxes, was proposed by Hryniewicz and Kaczmarek-Majer [5]. In their work, they used fuzzy control charts, proposed by Grzegorzewski and Hryniewicz [2], designed for monitoring the possibilistic index NSD, as described by Dubois and Prade [1]. In this paper, we propose an approach that unifies both aggragation methods. For monitoring purposes we propose a fuzzy control chart based on the credibility index Cr, proposed by Liu and Liu [6]. The statistical characteristics of control charts designed using this approach are evaluated through extensive simulation experiments.

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Relational modifiers corresponding to the Delphi method

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The Delphi method (see [2, 5], also known as the ETE (Estimate - Talk - Estimate) technique is based on the following principle: In the first round a panel of experts evaluates a proposal individually by all experts. Next, they announce and explain their evaluations. Finally, the evaluation is repeated, when all the experts may, but need not alter their original evaluations.

The expert decision can be represented, after a suitable normalization, as a mapping $f : X \to [0, 1]$, where X, the universe, is the set of all experts. Hence, an evaluation can be identified with a fuzzy set on the universe X. The change of the evaluation may be then understood as a result of a modification, as described in [4]. We can assume that the possible change of the evaluation in the second round of Delphi technique may be caused not only by the strength of the arguments while explaining the individual evaluations, but also by the strength of mutual acceptance among the experts. This can be represented by a fuzzy relation on the set X^2.

Modifiers accounting the relation among the elements of the universe, known as the relational modifiers, have been introduced and studied in [1] and [3]. In these works the relational modifier of the mapping f is given by the formula

$$M_R^C(f)(x) = \sup\{C(R(x, y), f(y)), y \in X\},$$

where C is a conjunctor on the unit interval.

However, this attitude does not completely correspond to the idea of the Delphi method. Namely, if we assume the reflexivity of R (which is indeed a natural assumption - an evaluator should be personally consistent), we come to an expansive modifier, i.e. $M_R^C(f)(x) \geq f(x)$ for all $x \in X$ and an arbitrary conjunctor C. So, the result of the evaluation in the second round is at least as good as in the first one. Clearly, this does not reflect real situations.

The aim of our research is to provide a class of relational modifiers that are not necessarily expansive or restrictive (a dual property of expansivity) using suitable aggregation functions, namely OWA operators, and study their properties.

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Some properties of Zadeh's extension regarding d_p metrics

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Let X be a metric space, we denote by $\mathcal{F}(X)$ the family of all normal upper semicontinuous fuzzy sets of X with compact support. In this work we analyzed some properties of Zadeh's extension of a continuous function and a semiflow regarding d_p-metrics. It is proved that, if X is compact and $f : X \to X$ is a continuous function ($f : R \times X \to X$ is a semiflow), then its Zadeh's extension $\widehat{f} : \mathcal{F}(X) \to \mathcal{F}(X)$ ($\widehat{f} : R \times \mathcal{F}(X) \to \mathcal{F}(X)$) is continuous regarding to the d_p metric, for any $p \geq 1$. Then we study positive Lyapunov stability, contractibility and strong sensitivity of Zadeh's extension regarding d_p metrics.

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Assessing the stability of fuzzy linguistic summaries for time series

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Fuzzy linguistic summaries enable the description of large numeric datasets in natural language. Although their practical potential has been demonstrated with multiple applications for various domains including healthcare, it is still challenging to effectively monitor the sequences of fuzzy linguistic summaries, and thus, support, for example, remote health monitoring.

Previous research has explored some aspects of the interpretability of the sets of linguistic summaries [1]. However, the majority of the related work focuses primarily on evaluating the quality of individual sentences such as the degrees of truth, confidence, support, informativeness, or focus. Assessment of the quality of the groups of summaries becomes even more complex for real-world scenarios in which additional data becomes available over time or the existing information is incomplete.

In this contribution, we address the stability analysis of the sequences of fuzzy linguistic summaries (FLS) [2]. We consider the following general form of fuzzy linguistic summaries:

$$Q\ R_1 \star \ldots \star R_k\ y\text{'s are } P_1 \diamond \ldots \diamond P_l,$$

where $\star, \diamond \in \{\text{AND, OR}\}$, Q is a linguistic quantifier, $R_1, \ldots, R_k$ are qualifiers and $P_1, \ldots, P_l$ are summarizes.

The stability analysis of FLSs is approached with the statistical process control confronted with the fuzzy set-based knowledge representation of time [3].

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Internal pseudo-uninorms on bounded lattices

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Internal functions are characterized by the fact that the value is one of the inputs, i.e., if $I : X^n \to X$ is internal, then $I(x_1, \ldots, x_n) \in \{x_1, \ldots, x_n\}$, which makes them interesting in many areas such as decision making and many others. Assuming the associativity of an internal binary function I, we can show that I can be decomposed via Clifford's ordinal sum into the semigroups which are either trivial or projections onto one of the variables. Conversely, any Clifford's ordinal sum of projections to one of the variables and trivial semigroups always results in an internal associative function, and thus the characterization is complete.

In addition, we want to study the connection between internality, associativity, and monotonicity with respect to a given order. In this case we can divide internally associative non-decreasing functions into the following classes, which correspond to a semigroup having the highest index in the ordinal sum.

- Internal pseudo-uninorms, if the semigroup is trivial.
- Internal left uninorms[3], if the operation of the semigroup is projection to the second coordinate.
- Internal right uninorms[3], if the operation of the semigroup is projection to the first coordinate.
- Other, if the Clifford's ordinal sum is unbounded.

Focusing more on internal pseudo-uninorms, it was shown in [1, 2] that idempotent (pseudo)-uninorms on chains coincide with internal (pseudo)-uninorms. This is no longer the case for arbitrary lattice. On every lattice it is possible to define idempotent pseudo-uninorms, but there are lattices on which no internal pseudo-uninorm exists regardless of the choice of neutral element. We succeed in characterizing internal pseudo-uninorms on the bounded lattice, and we present the sufficient (and necessary) conditions for a lattice to possess an internal pseudo-uninorm with a given neutral element.

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Bipolar Sugeno integral and bipolar OWA operators on complete distributive lattices

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Motivation for this research is the paper by Lizasoain and Moreno [5]. In that paper, the authors introduced OWA operators on complete lattices and they showed that the OWA operators on complete distributive lattices coincide with a kind of discrete Sugeno integral.

Kalafut and Kalina in [4] introduced unifunctions on $[0,1]$ which have an idempotent element $e \in [0,1]$, on $[0,e]^2$ the unifunction coincides with an overlap function (see [1]), on $[e,1]^2$ the unifunction coincides with a grouping function (see [2]) and otherwise it is commutative monotone and continuous. Unifunctions can be defined also on lattices, just continuity has to be replaced, e.g., by intermediate value property.

As a first step for introducing bipolar OWA operators, we need to define a kind of bipolar Sugeno integral. Let us have a bounded lattice $(L, \leq)$ and its dual $(L^d, \leq^d)$ where $(L^d, \leq^d)$ is a copy of $(L, \leq)$, just the order is reverted. Assume weighting vectors $(w_1, \dots, w_n)$ and $(w_1^d, \dots, w_m^d)$ fulfilling

$$\bigvee_{i=1}^{n} w_i = 1, \quad \bigwedge_{j=1}^{m} w_j^d = 1^d.$$

Then we may have a discrete Sugeno integral of inputs from L with respect to $(w_1, \dots, w_n)$, and a dual discrete Sugeno integral with respect to $(w_1^d, \dots, w_m^d)$ of inputs from L^d. Using a unifunction we can compute one value out of the two Sugeno integrals. The bipolar Sugeno integral is a kind of lattice-valued counterpart of the bipolar Choquet integral (see [3]).

To define a bipolar OWA operator on lattices, we must somehow order all inputs, more precisely, we have to order ”their absolute values". Elements of L are considered as non-negative and elements of L^d as non-positive. Particular weights should be assigned to the ordered values. Lizasoain and Moreno [5] used instead of the original input vector, where also incomparable elements might occur, a chain with the least element equal to the meet of all inputs and the greatest element equal to their join. We will also use that kind of chain, just we have to know which element of the chain corresponds to which element of the original inputs. We will consider a map τ that will contain the necessary information.

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Selection of loss function in Semi-Supervised Fuzzy c-Means

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Semi-Supervised Fuzzy c-Means (SSFCMeans), originally introduced in [1], extends the classical unsupervised Fuzzy c-Means (FCM) by incorporating partial supervision in the form of categorical labels assigned to a subset of observations. The primary objective of this approach is to leverage partial supervision to enhance clustering performance.

SSFCMeans employs an additive combination strategy, where the standard FCM objective function is augmented with a supervised component. This design choice has since become a widely adopted framework in various subsequent models (see [2] for a detailed literature review). The composite objective function of SSFCM is defined as:

$$J_{\text{SSFCM}} = \sum_{j=1}^{N} \sum_{k=1}^{c} u_{jk}^2 d_{jk}^2 + \alpha \cdot \sum_{j=1}^{N} \sum_{k=1}^{c} b_j (u_{jk} - f_{jk})^2 d_{jk}^2, \qquad (1)$$

where u_{jk} denotes the membership of the j-th observation in the k-th cluster ($j = 1, \ldots, N$; $k = 1, \ldots, c$), $d_{jk}^2 = d^2(x_j, v_k)$ represents the distance between the j-th observation and the k-th cluster prototype, b_j is an indicator variable set to 1 if the j-th observation is supervised and 0 otherwise, and $f_{jk} \in \{0, 1\}$ encodes the partial supervision. Finally, $\alpha > 0$ is the scaling factor that regulates the impact of partial supervision — the larger the α, the greater its impact on clustering outcomes.

The scaling factor α is commonly regarded as the primary hyperparameter controlling the extent to which partial supervision impacts model outcomes compared to fully unsupervised learning. Its role has recently been analyzed within a novel explainability framework [2].

However, α is not the only mechanism regulating the impact of partial supervision. In this paper, we provide new insights into the penalization term

$$(u_{jk} - f_{jk})^2$$

used in Eq. 1. We propose interpreting this term as a loss function, $\mathcal{L}(u_{jk}, f_{jk})$, which jointly shapes the effect of partial supervision along with the scaling factor α. In particular, we explore the potential of alternative loss functions, specifically considering the Hampel family of functions, which has been widely studied in robust statistics.

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Entropy measures for hesitant fuzzy sets

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The study of entropy measures in the fuzzy set theory can be traced to De Luca and Termini in 1972. The main aim was to quantify the uncertainty associated to a fuzzy set. An important family of fuzziness measures was given in Loo's definition [3]. This concept has been adapted to other types and extensions of fuzzy sets, such as Atanassov's IF sets [1], interval-valued fuzzy sets [2] or even hesitant fuzzy sets [4]. In our contribution we propose the entropy measure adapted to the hesitant fuzzy sets (HFSs).

Let $f_F : \mathcal{F}_H \to [0,1]$ be a mapping, $A, B \in \mathcal{F}_H$ and let $E \in \mathcal{F}(X)$ be the **equilibrium set**, i.e. $E(x) = 0.5$ for all $x \in X$. Therefore, E is the maximal fuzzy set, as the ambiguity of its membership or nonmembership is the largest as possible. The map f_F is said to be a **fuzziness entropy measure** associated to a hesitant divergence measure D_H if it satisfies the following properties:

(1) $f_F(A) = 0$ if and only if A is crisp, i.e. $A(x) \in \{0,1\}$ for all $x \in X$,
(2) $f_F(A) = 1$ if and only if $A = E$, $f(E)$ takes the maximum value of f since E is the equilibrium fuzzy set,
(3) $f_F(A) = f_F(A^c)$,
(4) $f_F(A) \leq f_F(B)$ if $D_H(A,E) \geq D_H(B,E)$.

Let $A(x) = \{h_1, \ldots, h_n\}$ be a general hesitant fuzzy element, $l_A(x)$ be its length and $\mathbb{H} \subseteq \mathcal{P}([0,1])$ be the set of all finite non-empty subsets of the closed interval $[0,1]$. We define a **score function** $s : \mathbb{H} \to [0,1]$ as follows $s : A(x) \mapsto \frac{1}{l_A(x)} \left[\sum_{h \in A(x)} h\right]$.

Some notions related to a hesitant fuzzy element $A(x)$ are presented: the lower bound $A^-(x) = \min \{h \mid h \in A(x)\}$ and the upper bound $A^+(x) = \max \{h \mid h \in A(x)\}$.

Moreover, we define a **difference function** $\Delta : \mathbb{H} \to [0,1]$ as follows $\Delta : A(x) \mapsto 1 - A^+(x) + A^-(x)$.

We present two other ways to construct a fuzziness entropy measure associated to a local hesitant divergence measure [2] based on the score and difference functions.

Let $Z : [0,1] \to [0,1]$ be a strictly decreasing real function and D_H be a local hesitant divergence measure with the following properties:

- $h(a, 0.5) = 0 \Leftrightarrow a = 0.5$,
- $h(a, 0.5) = 0.5 \Leftrightarrow a \in \{0, 1\}$,
- $h(a, 0.5) = h(1 - a, 0.5)$ for all $a \in [0, 1]$.

Accordingly, the function $f_F(A) = \frac{Z(2 \cdot D_H(A,E)) - Z(1)}{Z(0) - Z(1)}$ for a hesitant fuzzy set A is a fuzziness entropy measure based on the corresponding divergence D_H. In many practical cases we can consider the following three examples of the function $Z : [0,1] \to [0,1]$:

- $Z(t) = 1 - t^{\lambda}$, where $\lambda > 0$,
- $Z(t) = \frac{1-t}{1+t}$,
- $Z(t) = 1 - t \cdot e^{t-1}$.

Let $\Phi : [0,1] \to [0,1]$ be a strictly monotone decreasing real function with $\Phi(0) = 1, \Phi(1) = 0$ and D_H be a local hesitant divergence measure with the boundary condition $h(1, 0) = 1$. Then $f_F(A) = \Phi\left(D_H(A, A^c)\right)$ is a fuzziness entropy measure based on the corresponding divergence D_H.

In the next step, we introduce the second type of entropy measure on HFSs, the **hesitance entropy measure**. Finally, we present the **joint fuzziness-hesitance entropy measure**. Our study is completed with several examples.

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Impact of data preprocessing methods on the performance of machine learning models

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In machine learning, we often aim for the best performance, and this is reflected not only in the choice of an adequate algorithm but also in the preprocessing of input data [1]. We can consider many data transformation approaches, from basic operations such as filling by mean data gaps to more complex fuzzy-based methods [2], [3]. However, testing many different approaches is associated with high computational complexity and model training time. Learning curves provide insights into the dependence of performance on the data set size [4]. In addition, they can be used for model selection, predicting the impact of more training data on the quality of trained models, or selecting optimal hyperparameters [5]. However, the question arises as to how much we can trust the obtained results and whether the obtained results are transferable to new, previously unanalyzed data sets. In addition, domain knowledge is essential in selecting the best data processing methods for modeling. Fuzzy logic has been successfully used to incorporate expert knowledge into decision-making processes. This technique can make machine learning more comprehensible, as it can help combine natural language rules with numerical decisions.

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Dispersed Data Classification with Rule Induction and Conflict Analysis

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Modern systems store data in a dispersed manner, hindering classification due to the lack of a unified structure and inconsistencies. Approaches, such as federated learning [3] or ensemble methods [4], do not enable recognition of conflicts between independent data sources. This requires the use of specialized integration and processing methods that allow for comprehensive analysis and improved classification quality.

To address these challenges, we propose a classification method for dispersed data that applies Pawlak's conflict analysis model [2]. The method facilitates the collaboration of local decision tables with similar data, integrating them into coalitions. Aggregation relies on statistical characteristics of conditional attribute values. Each decision table is represented as

$$D_i = (U_i, A, d), i \in \{1, \dots, n\}, \tag{1}$$

where U_i is a set of objects, A is a common set of conditional attributes, and d is the decision attribute. For numerical attributes, the mean value is computed for each local table, along with the global mean and standard deviation across all local tables. The attribute is then assigned a value of 1, 0, or -1 based on its relation to the global values. For categorical attributes, a frequency vector of value occurrences is created for each local table. The tables are then grouped using the 3-means clustering method and assigned a value of 1, 0, or -1 according to value distribution similarity. Based on this, the conflict function is defined to measure the intensity of differences between pairs of tables as

$$\rho(D_i, D_j) = \frac{card\{a \in A : a(D_i) \neq a(D_j)\}}{card\{A\}}. \tag{2}$$

Tables with a conflict value below a predefined threshold form a coalition, within which they cooperate and construct an aggregated table.

For each coalition, a set of decision rules is constructed using four induction strategies: the exhaustive algorithm, the covering algorithm, the genetic algorithm, and LEM2. These rule sets serve as local classification models, with the final decision made by majority voting. A test object is classified using one of three approaches. In the first approach (FA), the decision class of the first matching rule is assigned. In the second (SA), the most frequent decision class among applicable rules is selected, with ties resolved randomly. In the third (TA), each covering rule is weighted by its match count relative to the total objects in the aggregated table linked to the coalition. The final class is chosen based on the highest cumulative weight. If no rule covers the test object, the decision class is chosen randomly.

The experiments were conducted on two data sets from the UCI Machine Learning Repository [1]: Vehicle Silhouettes and Car Evaluation. The data was split into a training set (70%) and a test set (30%), and the training set was stratified into 5, 7, 9, and 11 local tables. Classification performance was evaluated using accuracy and weighted F1-score. The results were compared with a baseline approach, where decision rules were generated separately for each local table, and the final decision was made based on majority voting.

The results show that classification quality differs across data sets. In Vehicle, SA and TA outperformed FA in F1-score, suggesting multiple rules enhance reliability. Conversely, Car exhibited stable performance across all methods, implying lower sensitivity to rule induction strategies. A higher number of coalitions suggests a more dispersed data structure, potentially reducing effectiveness. The baseline and proposed approaches performed similarly. The inconsistent results across induction strategies highlight the need for further refinement to improve generalization and classification robustness.

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Different approaches to defining the union and intersection of balanced fuzzy sets using uninorms and nullnorms

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In the context of fuzzy set theory and operations, nullnorms and uninorms are generalizations of the classical t-norms and t-conorms. These operations are relevant in the study of balanced fuzzy sets and their union and intersection operations.

A balanced fuzzy set is typically defined (see [2, 4]) in such a way that its union and intersection operations exhibit symmetry between the membership functions of the fuzzy sets involved. The definition of a balanced fuzzy set A in X has the following form:

$$\eta_A(x) = \begin{cases} \mu_A^1(x), & x \text{ in set } X, \\ 0, & \text{when } x \text{ is in and out the set in the same degree} \\ \mu_A^2(x), & x \text{ not in set } X, \end{cases}$$

where $\mu_A^1(x) \in (0,1]$ and $\mu_A^2(x) \in [-1,0)$, where $x \in X$.

Nullnorms and uninorms extend the idea of t-norms and t-conorms, respectively. These operations are designed to handle fuzzy set operations in more general forms. A nullnorm is an operation that allows the zero element to lie within the whole interval $[0,1]$, while in the case of a uninorm, this applies to the neutral element (for more details see [1, 5]).

The union of two fuzzy balanced sets A and B having, respectively, the membership functions η_A and η_B is the set $C = A \cup B$ having the membership function described by the formula:

$$\eta_{A\cup B}(x) = SB(\eta_A(x), \eta_B(x)), \quad \text{for} \quad x \in X.$$

where SB is balanced conorm.

The intersection of fuzzy balanced sets A and B having, respectively, the membership functions η_A and η_B is the set $C = A \cap B$ having the membership function described by the formula:

$$\eta_{A \cap B}(x) = TB(\eta_A(x), \eta_B(x)), \quad \text{for} \quad x \in X.$$

where TB is balanced norm.

In this work, we aim to use uninorms and nullnorms to construct balanced t-norms and t-conorms. In the case of nullnorms and uninorms, the behavior of union and intersection can be adjusted to account for more flexibility or different types of interaction between the sets. These operations provide a generalized framework for these interactions, where the union and intersection might behave differently depending on the properties of the nullnorm or uninorm used.

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A Three-Valued Logic Approach to Data Mining in Survey Research

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Observation of the surrounding world and the need to process data have influenced the development of logic, starting from classical, Łukasiewicz's three-valued logic and fuzzy logic. In Łukasiewicz's logic [2], the value 0.5 represents the state of possible truth. This allows modeling uncertainty and an intermediate state.

Based on three-valued logic, a new method is presented that takes into account the problem of analyzing incomplete data and the uncertainty of human decisions by modeling the intermediate state between truth and falsehood.

Łukasiewicz's three-valued logic is isomorphic to logic with the values $\{-1, 0, 1\}$, when applying the bijection $f(x) = 2x - 1$, which maps the standard truth values 0, 0.5, 1 to -1, 0, and 1, respectively. After that the logical operators are defined as follows: negation $N(x) = -x$, conjunction $C(x, y) = \min(x, y)$, disjunction $D(x, y) = \max(x, y)$, and implication $I(x, y) = \min(1 - x + y, 1)$. In the presented approach, the intuitive meanings of 1 as truth, -1 as falsehood, and 0 as information deficiency are used.

This method was used to analyze an actual conditional branching questionnaire in a true-false format. These questionnaires are characterized by the presence of many missing values. Several approaches can be used to address this issue, such as data imputation or data mining methods designed for datasets with many missing attribute values [1, 7]. However, to determine whether a relationship exists between the indicated attributes and the decision, a computationally simpler and more transparent approach involves the use of trivalent logic.

The proposed method includes the following steps:
Step 1. Data are entered in matrix form R.
Step 2. Implications are determined for the corresponding values of

the vectors R^a and R^d, i.e.: $i_j = I(R^a_j, R^d_j)$, where a and d are the column numbers indicated by the expert as attribute and decision.
Step 3. The percentage of implications that are true for all outcomes p is examined.
Step 4. The test accuracy level $\epsilon \geq 0$ is determined.
Step 5. The relationship between the indicated premise and the conclusion is verified by the condition $p + \epsilon \geq 1$.

As an additional result of this method, relations can be presented in the form of an incidence matrix A, which indicates the dependencies between attributes; thus, data dependencies can also be presented using graphs.

When the Likert Scale [3] is used in surveys, the results obtained can be fuzzified, and research can be carried out using fuzzy sets [8]. Recall that in order to study the relationship between attributes and the decision, one can use fuzzy relational equations [4–6]. The presented method can be extended to fuzzy sets in the future. Such an extension will allow for determining the relationship between the indicated attributes and the decision.

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Consistency of fuzzy linguistic summaries

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In this contribution we present the definition of fuzzy linguistic summaries (FLS) [2]. They are sentences that describe huge numerical data sets in natural language [3]. Our approach include complex forms of such summaries of the following form:

$$Q\ R_1 \star \ldots \star R_k\ y\text{'s are } P_1 \diamond \ldots \diamond P_l, \tag{1}$$

where $\star, \diamond \in \{\text{AND, OR}\}$, Q is a linguistic quantifier, $R_1, \ldots, R_k$ are qualifiers and $P_1, \ldots, P_l$ are summarizers. Moreover, we focus on the consistency of FLSs which consists of double negation property and non-contradiction property [1]. We present conditions for negations of linguistic quantifiers and summarizers that allow consistency (for selected FLSs).

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HuReTEx: Human Readable Twin Explainer for Deep Learning Models Based on Imprecise Information Flow Models - a General View

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One of the most current problems in Artificial Intelligence (AI) is to make AI tools human-readable (interpretable, explainable, etc.) and in consequence to make AI responsible (cf. [1]). AI tools are currently, in many cases, reinforced by deep neural networks (DNNs). Therefore, special attention in research on Explainable Artificial Intelligence (XAI) is focused on Explainable Deep Learning (XDL) (cf. [5]). Taxonomy of the trends identified for explainability techniques related to Deep Learning Models (DLMs) distinguishes, among others, model-agnostic techniques (MATs) and model-specific techniques (MSTs) (cf. [1]). An explainer in MATs is able to explain any model (cf. LIME technique [6]). An explainer in MSTs is correlated with a given DLM (cf. DeepLIFT technique [7]). In our research, we are developing a new technique that can be classified as MSTs. This technique is provided with an acronym HuReTEx (Human Readable Twin Explainer for Deep Learning Models). An idea of transformation of a deep learning model (DLM) into imprecise information flow model (IIFM) via a Sequential Information System (SIS) is shown in Figure 1(a). In this transformation, DLM is original, numerical and machine-readable, while IIFM is a twin of DLM that is symbolical and human-readable. HuReTEx can be treated as reference to the ideas of mirror worlds (cf. [2]) as well as digital twins (cf.

[3]). For a model that is unreadable to humans, its readable twin is built. Transformation is preformed in several main stages (see a

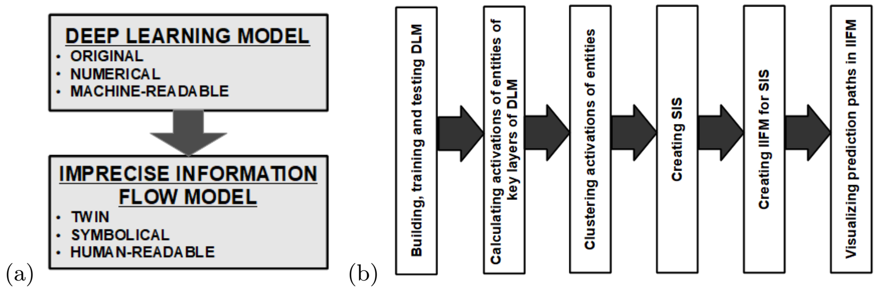

Fig. 1. (a) An idea of transformation of DLM into IIFM, (b) A flowchart of the transformation procedure of DLM into IIFM.

flowchart in Figure 1(b)). Rough set flow graphs (RSFGs) [4] (used as IIFMs) and triangular norms or co-norms together with genetic algorithms can be used to mine and visualize the most confident prediction paths in RSFGs explaining decisions proposed by DLMs.

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A Privacy-Focused Decision Support System for Advanced Medical Diagnostics Using Federated Learning

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The study tackles the challenge of developing diagnostic models while safeguarding data privacy. This issue is especially critical in medical applications like breast cancer diagnosis, where individual organizations often lack sufficient or high-quality data to train robust models and cannot share their datasets due to privacy regulations.

Federated learning [1–3] offers a viable solution by allowing multiple entities to train a machine learning model collaboratively without exchanging raw data, thereby preserving privacy. Our research examines horizontal federated learning, in which each participating institution refines its local model iteratively. These updates are periodically aggregated and redistributed to improve the overall model's effectiveness. The aggregation process typically involves weighted averaging, with different strategies used to determine the weights.

In this paper, we investigate federated learning in the presence of uncertainty by incorporating the Choquet integral as an information fusion technique while extending the research undertaken in this direction in [4, 5]. We integrate uncertainty measures, such as entropy, into the aggregation process and analyze local model parameters to understand how their performance and data uncertainty influence the learning process.

This study focuses on two key aspects of modern decision support systems: federated learning and uncertainty modeling. Specifically, we explore:

- Federated learning as a method for managing uncertain and/or incomplete data, particularly using interval-valued fuzzy sets.
- Fusion techniques that enhance local model performance, with a novel application of the Choquet integral incorporating entropy-based uncertainty measures.

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Modeling and decision-making methods with ordered fuzzy numbers

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In the contemporary context of information systems supporting decision-making processes in operational management of enterprises, there is an increasing necessity for these systems to operate under conditions of significant uncertainty. Such uncertainty can arise from dynamic environmental fluctuations, incomplete or imprecise data, and the necessity of adapting to unpredictable circumstances. In such contexts, traditional deterministic methods often prove insufficient, prompting the adoption of probabilistic and fuzzy approaches. However, both fuzzy models and probabilistic-fuzzy models [1] do not directly incorporate the dynamics of variables, which constitutes a critical issue, particularly in light of the rapid pace of change in technological, business, and social environments. Consequently, the necessity arises to construct models for complex problems of analysis and synthesis in discrete event systems. This necessitates the consideration of a broader range of information than that permitted by the notation of classical convex fuzzy sets [2] and Mamdani-type fuzzy systems, as well as the efficient arithmetic operations that avoid the undesirable expansion of uncertainty.

The primary objective of this paper is to demonstrate that, through proper data preparation, appropriate model selection, and its structure, in conjunction with domain knowledge in management, production engineering, and logistics, fuzzy knowledge modeling and data analysis methods utilizing Ordered Fuzzy Numbers (OFNs) [3] constitute effective, interpretable, and computationally efficient tools that support decision-making processes under epistemic uncertainty while accounting for the dynamics of decision-making factors.

The application of OFNs for knowledge modeling or data analysis in decision-making contexts allows for the consideration of various sources of uncertainty (e.g. incomplete knowledge, expert subjectiv-

ity, and qualitative data analysis) and, due to the OFN orientation, facilitates the incorporation of trends in the dynamics of decision-making factors. As a result, models employing OFNs are intuitive, easily interpretable, and less complex compared to traditional fuzzy models.
Empirical evidence from research and practical applications [4–6] substantiates the efficacy of approaches based on OFNs in conjunction with fuzzy modeling and inference, or suitable multi-criteria decision-making techniques. These approaches effectively support decision-making processes in complex, dynamic and uncertain conditions. Consequently, the implementation of such solutions in practice has the potential to enhance the efficiency of management, production and logistics processes, thereby facilitating more accurate and informed decision-making.

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Partial fuzzy logics – how far can we get in modeling the missing data in the knowledge systems

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Three-valued logics [1] belong to classical topics investigated by logicians since 1920's thanks to the seminal works of Jan Łukasiewicz [2]. He introduced the very first three-valued logic that showed that the standard binarity of truth and false needs not be the only setting and that extended sets of truth values may offer a bigger flexibility in modeling distinct phenomenons. This brilliant idea was followed by many scholars, for instance Bochvar [3] introduced a three-valued logic where the third truth value, typically a dummy value $\star$, represents an unrepairable critical system error that stops the calculation. Another approach introduced by Sobociński [4] deals with the value $\star$ as it was a neutral element and this, it mimics the situation when the third value is supposed to be ignored.

Not surprisingly, many such three-valued logics were extended to partial fuzzy logics that allow modeling distinct types of undefined truth-values, even in the graded sense. Such logics and related algebras may model reasoning with non-denoting terms, missing or unknown values, and other interesting cases. However, in order to be able to model real cases, the algebraic models need to be mirrored in applied tools such as decision-making or any other knowledge systems.

In such systems, there are two key issues to be taken into account. First of all, the dummy value and the related algebra (logic) should be constructed in a way that is appropriate for modeling distinct types of, e.g., frequently occurring N/A values, while the case of Bochvar and Sobociński are undoubtedly logically sound however, from the application point of view they have limited potential. Secondly, the knowledge systems based on rely on distinct algebraic properties and in principle certain quality is guaranteed if the operations preserve some well-defined algebraic structure form, e.g., MV-algebra, BL-algebra, or at least residuated lattice. However, algebras for partial fuzzy logics do not preserve all the properties [5] and thus, they do not constitute residuated lattices.

The first issue was approached by building algebras that adopted the lower bound view on the meaning of the dummy value $\star$ which showed a convincing potential in modeling the missing values problem in decision/making or classification tasks, see e.g. [6]. However, the residuated algebra has not been reached. Interestingly, if an analogous approach was employed dually to come up with a sort of upper boundary strategy, the residuated lattice structure was obtained.

The natural question that arises is, whether we may get back to lower boundary strategies and design an algebra that would also reach the residuated lattice quality while keeping the applicationally well-supported model of the missing values. This talk presents a contribution that elaborates this question.

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On fuzzy threshold in measurement theory

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Measurement is described as a homomorphism Φ of an empirical structure into a mathematical structure [3, 7]. The empirical structure is equipped with an order relation $\prec$ and a binary operation (see ex. [1]). For the fuzzy model, in the paper [9], we proposed a fuzzy order in fuzzy sets based on t-norm fuzzy arithmetic (see also [8]). The definition of the order comes from the extension principle for interval order: $a > b$ if and only if $a - b > 0$. In measurement theory [2, 4–6], the order is defined using the two-argument threshold function $\delta(a, b)$: $a \prec b \iff \Phi(a) + \delta(a, b) < \Phi(b)$.

The goal of this study is to determine the conditions under which a fuzzy threshold exists that satisfies the equation $B \ominus A > \delta(A, B)$.

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Soft labeling in semi-supervised hidden Markov models

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Hidden Markov Models (HMMs) are widely used latent-variable models that provide flexible approaches to learning from sequential data. However, standard HMM training typically relies on either fully supervised or fully unsupervised methods, both of which can be suboptimal in many real-world settings where data labeling is often expensive or imperfect. A solution proposed by [**?**] integrates partial supervision into HMMs by modifying the forward–backward recursions, effectively constraining certain paths in the state lattice when partial labels are known. While this approach elegantly handles hard labeling constraints, it does not extend naturally to scenarios with soft or uncertain labels because the forward–backward definition must remain consistent across all time steps. In this work, we propose a novel semi-supervised HMM training strategy that relocates the burden of partial labeling from the forward–backward variables to the emission probabilities. By incorporating partial supervision directly into the emission densities, we preserve the standard HMM recursion while leveraging soft labeling information as a prior distribution. On top of that, we are able to recover the original approach when only hard labeling is used. Our method broadens HMM applicability to settings where label uncertainty must be carefully accounted for in model inference. Consequently, it naturally accommodates cases in which labels are not entirely reliable: whether due to subjective human annotation or automated labeling processes prone to error. In controlled experiments with simulated data, we examine how varying structural assumptions (particularly the extent of overlap among emission distributions) interact with soft labeling to produce different outcomes. We demonstrate that, in many cases, applying soft labels to mislabeled data can significantly enhance a model's predictive performance.

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A feature selection method based on aggregation functions

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Feature selection [2] is important in high-dimensional datasets to identify the most relevant features [3, 5]. This process reduces dimensionality and improves model performance. Feature selection methods are usually described as filter (correlation-based), embedded (integrated within learning algorithms), or wrapper (evaluated externally). We introduce Interval Valued Weighted Feature Ranking ($IVWFR$), a novel method aggregating interval-valued feature importance measures across multiple data splits. This approach incorporates aggregation functions [1] and interval modeling [6] to capture uncertainty in feature relevance. Experiments show that $IVWFR$ outperforms well-known methods such as RFE, RFECV, Gini Index, Mutual Information, and Random Forest rankings, particularly on datasets which are hard for classification.

$IVWFR$ enhances feature-importance scores by computing feature importance intervals through stratified cross-validation and aggregating them into representative intervals.

The steps of IVWFR are as follows:

- Creating n stratified folds to maintain class balance.
- Training a classifier on each fold and extracting feature importances.
- Computing importance intervals.
- Applying weighted interval-valued aggregation and ranking.
- Selecting features based on the chosen method while maintaining a minimum feature count.

For each feature with multiple confidence intervals, we compute a ranking score based on importance (interval center) and stability (interval width). Both values are min-max normalized to [0,1], with stability inverted, as a result narrower intervals receive higher scores. The final ranking is derived using aggregation functions such as arithmetic, geometric, or harmonic means, balancing importance and stability through weighting parameters.

The *IVWFR* method was evaluated on 10 synthetic datasets [4]. Synthetic datasets contain relevant, redundant, and irrelevant features. Feature-importance intervals were aggregated, producing a final ranking validated by classification accuracy and feature recognition rates. Compared to RFE, RFECV, Gini Index, Mutual Information, and Random Forest, *IVWFR* achieved higher accuracy and superior feature identification, with notable improvements on complex datasets (e.g., Double Spiral, 5D XOR described in [4]). Moreover, well-known real-world microarray datasets were used for further evaluation.

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A Fuzzy Set-based Approach to Evaluating the Value of Information Under Uncertainty

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In many real-world decision-making scenarios, achieving the optimal outcome is conditioned upon determining key parameters. However, as these parameters are typically uncertain, the optimal result cannot be achieved. To deal with this uncertainty, decision-makers (DM) may undertake additional investigations, which incur associated costs. When following this line, it is crucial to assess whether the potential benefits of acquiring more information justify the costs. This assessment is formalized through the Value of Information (VoI), which quantifies the advantage of obtaining further insights before making a decision.

Specifically, given an objective function $v(x, u)$, where x is a random variable and u is the action, VoI is computed as the difference between the two values:

$$\mathrm{VoI} = \mathbb{E}_x\big(\max_u v(x,u)\big) - \max_u \mathbb{E}_x\big(v(x,u)\big).$$

The first one describes the expectation of the optimal profit under the condition that the DM knows the realization of the random variable x, while the second refers to the situation, where the DM cannot observe the realization and hence optimizes the averaged problem.

Although VoI has gained increasing popularity in recent decades (see, e.g., [1–3]), this approach has its drawbacks. Its applicability can be severely restricted in situations where modeling the problem probabilistically is difficult or infeasible, such as when the probability distribution of a key variable is unknown, cannot be reliably estimated, or when no probability exists due to the nature of the problem.

To overcome this difficulty, we propose an alternative approach that involves fuzzy sets to handle uncertainty, where in the classical case, a probability distribution is used.

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